

UG — Math

2020

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any four questions in which

Q. No.1 is compulsory.

1. Answer all questions : 2×10 = 20

(a) Find the solution of the partial differential equation $px = y$.

(b) Find the value of the differential, $d\left(\frac{y}{x}\right)$.

(c) Find the integrating Function for the differential equation $\frac{dy}{dx} + 1 = e^{x-y}$.

(d) Write the Clairaut's form equation.

(e) Solve $p - 1 = 0$.

(f) Find the solution of $\frac{d^2y}{dx^2} - 4y = 0$.

(g) Find the degree of the differential equation

$$\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = k \frac{d^2y}{dx^2}$$

(h) Write the conditions for exactness of the differential equation $Pdx + Qdy + Rdz = 0$.

(i) Find the Complementary Function for the following differential equation :

$$\left(x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x\right)$$

(j) Form a partial differential equation by eliminating a from $y = x + a$.

2. (a) Solve $x \frac{dy}{dx} = y + \cos \frac{1}{x}$ 10

(b) Solve $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y}{x^2} (\log y)^2$ 10

3. (a) Solve $x \left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$. 10

(b) Solve $y = (1+p)x + ap^2$. 10

4. (a) Find the complete solution of

$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$. 10

(b) Solve $\frac{d^2y}{dx^2} + a^2y = \cos ax$. 10

5. (a) Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$. 10

(b) Solve the differential equation, using variation of parameters $\frac{d^2y}{dx^2} + y = \sec x$. 10

6. (a) In usual symbols, prove that,

$\int_{-1}^{+1} P_m(x) P_n(x) dx = 0$ for $m \neq n$. 10

(b) Solve $n P_n(x) = x P_n'(x) - P_{n-1}'(x)$. 10

7. (a) For a Bessel function $J_n(x)$, prove that

$x J_n'(x) = -n J_n(x) + x J_{n-1}'(x)$. 10

(b) Prove that for a Bessel function $J_n(x)$,

$J_{-n}(x) = (-1)^n J_n(x)$. 10

8. (a) Find the Laplace transform of

$e^{-3t}(2 \cos 5t - 3 \sin 5t)$. 10

(b) Find $L^{-1} \left(\frac{4s+5}{(s-1)^2(s+2)} \right)$. 10

