UG — Math (C – 501)

2019

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer four questions in which Q. No. 1 is compulsory.

1. Answer all questions : 2×10 = 20

(a) Show that $\lim_{\substack{x\to 0 \\ y\to 0}}\left[\frac{2xy}{x^2+y^2}\right]$ does not exist.

(b) If $x = r\cos\theta$, $y = r\sin\theta$, show that $\left(\frac{\partial x}{\partial r}\right)\left(\frac{\partial r}{\partial x}\right) = \cos^2\theta$.

(c) If u be a composit function of t given by the relations $u = f(x, y)$, $x = \phi(t)$ and $y = \psi(t)$ then write the value of $\frac{du}{dt}$ and du.

(Turn over)

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(d) State sufficient condition of maxima-minima of a function of two variables.

(e) Evaluate $\iint_R x^2y\,dx\,dy$ over R where $R : \{0 \le x \le 1, 0 \le y \le 2\}$.

(f) Find Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ when $x = r\cos\theta$, $y = r\sin\theta$

(g) Define a vector field

(h) Define directional derivative of a function.

(i) Define curl of a vector function.

(j) Find grad ϕ if $\phi = \log(x^2 + y^2 + z^2)$.

Group – B

2. (a) Show that the function $f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 - y^2}; & (x, y) \ne (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ is continuous at (0, 0) 10

Contd.

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(b) If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2-u}+\frac{z^2}{c^2-u}=1$, prove that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=$$

$$2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial x}\right).$$ 10

3. (a) State and prove sufficient condition for differentiability of a function of two variables. 10

(b) Find all the maximum and minimum values of the function designed by $f(x, y) = x^3 + y^3 - 63(x + y) + 12\,xy$. 10

4. (a) Integrate $\iiint xyz\,dxdydz$ over the domain $x^2 + y^2 + z^2 \le 1$. 10

(b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\,dxdy$. 10

5. (a) Find the volume of the sphere $x^2 + y^2 + z^2 \le a^2$ where $a > 0$ by using triple integral. 10

(b) Change the order of integration in

$$\int_0^1 \int_0^{x(2-x)} f(x,y)\,dy\,dx$$ 10

6. (a) Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $p(1, 2, 4)$. 10

(b) Prove that curl $(\vec{a} \times \vec{b}) = (\vec{b} \cdot \nabla)\vec{a} - (\vec{a} \cdot \nabla)\vec{b} + \vec{a}\,\mathrm{div}\,\vec{b} - \vec{b}\,\mathrm{div}\,\vec{a}$. 10

7. (a) Show that $\nabla \cdot \left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$. 10

(b) Intregrate the function $\vec{F} \cdot d\vec{r}$, where $\vec{F} = x^2\hat{i} - xy\hat{j}$ from O(0, 0) to P(1, 1) in each of the following cases : 10

(i) Along the straight line OP

(ii) Along the parabola $y^2 = x$ 10

8\. (a) If $\vec{F} = 2x^2\hat{i} - 4xy\hat{j} + zx\hat{k}$, evaluate $\iint_s \bar{F} \cdot \hat{n}\, ds$ where s is the surface of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 1$. 15

(b) Give the statement of Stoke's theorem. 5