UG — Math (C – 602)

2021

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. **Answer all** questions of the following : 2×10 = 20

   (a) Define vector space.

   (b) Define subspace.

   (c) Define linear independence in a vector space.

   (d) Define linear transformation.

   (e) Define Null space.

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   (f) State the Rank Nullity theorem.

   (g) Define self-adjoint operator.

   (h) Define an inner product space.

   (i) Define Normal operator.

   (j) Define orthogonal complement.

2. Let F be an arbitrary field, n a positive integer and let $V_n(F)$ be the set of all ordered n-tuples of elements of F. That is the element of $V_n(F)$ are of the form $(a_1, a_2, a_3, \cdots\cdots, a_n)$; where each $a_n \in F$ then prove that $V_n(F)$ is a vector space over F. 20

3. (a) Show the necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V(F) is : $a, b \in F$ and $\alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$. 10

   (b) Examine whether the set of all such vectors $\alpha = (a_1, a_2, a_3, \cdots\cdots, a_n)$ in R^n where a_2 is reational, is a subspace of R^n. 10

4. State and prove the Extension Theorem. 20

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5\. (a) Let T be a linear transformation from a vector space U(F) into a vector space V(F) then show that : 20

(i) T(0) = 0, where 0 on the L. H. S. is a zero vector of u. and 0 on the R. H. S. is zero vector of V.

(ii) $T(-x) = -T(x)$ for all $x \in u$.

(iii) $T(x - y) = T(x) - T(y)$ for all $x, y \in u$.

(iv) $T(a_1x_1 + a_2x_2 + \cdots\cdots + a_nx_n) = a_1T(x_1) + a_2T(x_2) + \cdots\cdots + \cdots\cdots + a_nT(x_n)$

6\. (a) State and prove the Caley-Hamilton theorem. 10

(b) Verify Caley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -1 & -4 & -4 \end{bmatrix}$$ and use if to find A^{-1}.

10

7\. (a) Define inner product space and show that an inner product space function in a Hilbert space is jointly continuous. 10

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(b) Give an example of an incomplete inner product space. 10

8\. (a) If A_1 and A_2 are self-adjoint operators on H then their product A_1A_2 is self-adjoint *iff* $A_1A_2 = A_2A_1$. 10

(b) An operator T on H is self-adjoint *iff* (Tx, x) is real for all $x \in H$. 10

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