

UG — Math (D-603) A

2021

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any four questions in which

Q. No. 1 is compulsory.

1. Answer all questions of the following .

2×10 = 20

(a) If -4 is one root of the equation $2x^3 + 6x^2 + 7x + 60 = 0$. Find the other roots.

(b) If $\alpha, \beta, \gamma, \delta$ are the roots of the Eq. $x^4 + px^3 + qx^2 + rx + s = 0$, find $\alpha + \beta + \gamma + \delta$.

(c) If α, β, γ are the roots of the Eq. $x^3 + px^2 + qx + r = 0$, find $\alpha^2 + \beta^2 + \gamma^2$.

(d) State Newton's theorem on the sum of the powers of the roots.

(e) Change the sign of the roots of the equation $x^5 + 4x^3 + 7x^2 + 5 = 0$.

(f) An equation of the form $x^3 + 3Hx^2 + G = 0$, whose one root is real and other two roots are imaginary then find the sign of $G^2 + 4H^3$.

(g) Show that the equation $x^n - 1 = 0$ has only two distinct roots 1 and -1 when n is even.

(h) State fundamental theorem of algebra.

(i) Find the equation whose roots are two times of the roots of the equation $x^3 - 3x^2 + 5x + 8 = 0$.

(j) If $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ are the roots of the equation,

$a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n = 0$, then find $\alpha_1 \alpha_2 \alpha_3 \dots + \alpha_n$.

2. (a) If two roots of the equation $x^5 - x^4 + 8x^2 - 9x - 15 = 0$ are $\sqrt{3}$ and $(1+2i)$, find the other roots. 10

(b) Solve the equation $4x^4 - 28x^3 + 51x^2 - 7x - 20 = 0$, whose roots are in A. P. 10

3. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ are in G. P. 10

(b) Find the condition that the roots $\alpha, \beta, \gamma, \delta$ of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ should be connected by the relation $\alpha\beta = \gamma\delta$. 10

4. (a) Calculate the values of the following symmetric functions for the cubic equation $x^3 + px^2 + qx + r = 0$ whose roots are α, β, γ
(i) $\sum \alpha^2\beta^2$
(ii) $\sum \alpha^3$ 10

(b) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the terms of coefficient
(i) $\sum \alpha^2\beta^2$
(ii) $\sum \alpha^3$ 10

5. State and prove Newton's theorem on the sum of the powers of the roots. 20

6. (a) Solve the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$, by removing the second term. 10
(b) If α, β, γ , are the roots of the equation $x^3 + 8x$. Find the equation whose roots are $\beta + \gamma - \alpha; \gamma + \alpha - \beta; \alpha + \beta - \gamma$. 10

7. (a) Find the real roots of the equation $x^3 + 2x^2 - 51x + 110 = 0$ by Sturm's function. 10
(b) Solve the equation $x^3 - 30x - 133 = 0$, by Cardon's, method. 10

8. (a) Solve the biquadratic equation $x^4 - 3x^2 - 6x + 2 = 0$. 10
(b) State and prove Descarte's rule of sign. 10

