UG — Math (C – 410)

2020

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any four questions in which Q. No. 1 is compulsory.

1. Answer all questions : 2×10 = 20

(a) Define characteristic ring.

(b) Define ring without zero divisors.

(c) Define ring with zero divisors.

(d) Define subring.

(e) Define Ideal.

(f) Define Principal Ideal

(g) Prove in a ring $(R_1 +_i)$

$a \cdot 0 = 0 \cdot a = 0, \forall a \in R$

(h) Define factor ring.

(i) Define Kernel of a ring homomorphism.

(j) Prove $f(-a) = -f(a), \forall a \in R$ where $(R_1 +_i)$ is a ring.

2. (a) Define ring, integral domain and Field. 10

(b) Give an example of non-commutative ring with unit element. 10

3. (a) A ring $(R_1 +_i)$ is without zero divisors iff the cancellation law holds in R i-e for a, b, c $\in$ R with $a \pm o$:

(i) $ab = ac \Rightarrow b = c$

(ii) $ba = ca \Rightarrow b = c$ 10

(b) Show that a ring R in which $a^2 = a, \forall a \in R$ is commutative. 10

4. (a) Prove that every finite integral domain is a field. 10

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(b) Prove that intersection of two subrings is a subring. 10

5. State and prove fundamental theorem for homomorphism of a ring. 20

6. (a) at f is a homomorphism of a ring R into a ring R^1 with Kernel S then S is ideal of R.

10

(b) Let $F : R \to R^1$ is a homomorphism of a ring R into a ring R^1. at A is any ideal of ring R then f (A) is also and ideal of f (R). 10

7. Define :

(i) Vector space

(ii) Vector subspace

Prove that the set V_n (F) of order N-Tupples of elements of a field F is a vector space with respect to addition and scalar multiplication on $V_n(F)$. 20

8. (a) The necessary and sufficiant condition for a non-empty subset W of a vector space V(F) to be a subspace of V(F) is :

$a, b \in F$ and $\alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$. 10

(b) Let W_1 and W_2 be subspaces of vector space V. Then prove that $W_1 \cap W_2$ is a vector subspace of V. 10