UG — Math
(C – 203)

2020

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. Answer **all** questions : 2×10 = 20

(a) Define the set bounded above and the set bounded below.

(b) Define least upper bound and greatest lower bound.

(c) Define δ-neighourhood of a point in R.

(d) Define a sequence.

(e) Define limit of a sequence.

(f) Define convergent sequence

(g) State Bolzano Weierstras Theorem for sequences.

(h) For what value of p the series $\sum_{n=1}^{p} \frac{1}{n^p}$ is divergent ?

(i) Write Logarithmic Test.

(j) Define an absolutely convergent series.

2. (a) 'The real number system is everywhere dense but full of gaps.' Explain with an example. 10

(b) Show that a set S is a nbd. of p iff there exists a positive integer n such that $\left]p-\frac{1}{n}, p+\frac{1}{n}\right[\subset S$. 10

3. (a) Prove that every bounded set has a suprema and infima. 10

(b) Prove that between any two distinct real numbers there are infinitely many rathional numbers. 10

4. (a) Prove that Every Convergent Sequence is bounded. 10

(b) Prove that A monotonic increasing sequence bounded above tends to a limit which is its least upper bound. 10

5. (a) Prove that a Cauchy sequence of real numbers is bounded. 10

(b) Prove that the sequence $\sqrt{2}, \sqrt{(2\sqrt{2})}, \sqrt{\{2\sqrt{2(\sqrt{2})}\}}$, ——— convergens to 2. 10

6. (a) State and prove ratio test. 10

(b) Examine the convergency and divergency of the series $\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+$-----. 10

7. (a) State and prove Raabe's Test. 10

(b) Test the series whose general term is $\frac{\sqrt{n+1}-\sqrt{n}}{n^p}$. 10

8. (a) State and prove De Morgan's and Bertrand's Test. 10

(b) Show that the series :

$$1-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+-----+-----+(-1)^{n-1}.\frac{1}{n^p}$$

+ ----- is absolutely convergent for $p > 1$, conditionally convergent for $0 < p \le 1$ and divergent for $p \le 0$. 10