

2020

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any **four** questions in which Q. No.1 is compulsory.

1. Answer all questions : 2×10 = 20

(a) Find the solution of the partial differential equation  $px = y$ .

(b) Find the value of the differential,  $d\left(\frac{y}{x}\right)$ .

(c) Find the integrating Function for the differential equation  $\frac{dy}{dx} + 1 = e^{x-y}$ .

(d) Write the Clairaut's form equation.

(e) Solve  $p - 1 = 0$ .

(f) Find the solution of  $\frac{d^2y}{dx^2} - 4y = 0$ .

(g) Find the degree of the differential equation

$$\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = k \frac{d^2y}{dx^2}.$$

(h) Write the conditions for exactness of the differential equation  $Pdx + Qdy + Rdz = 0$ .

(i) Find the Complementary Function for the following differential equation :

$$\left(x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x\right)$$

(j) Form a partial differential equation by eliminating  $a$  from  $y = x + a$ .

2. (a) Solve  $x \frac{dy}{dx} = y + \cos \frac{1}{x}$ . 10

(b) Solve  $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y}{x^2} (\log y)^2$ . 10

3. (a) Solve  $x \left( \frac{dy}{dx} \right)^2 + (y-x) \frac{dy}{dx} - y = 0$ . 10

(b) Solve  $y = (1+p)x + ap^2$ . 10

4. (a) Find the complete solution of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x. \quad 10$$

(b) Solve  $\frac{d^2y}{dx^2} + a^2y = \cos ax$ . 10

5. (a) Solve  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$ . 10

(b) Solve the differential equation, using variation

of parameters  $\frac{d^2y}{dx^2} + y = \sec x$ . 10

6. (a) In usual symbols, prove that,

$$\int_{-1}^{+1} P_m(x) P_n(x) dx = 0 \text{ for } m \neq n. \quad 10$$

(b) Solve  $n P_n(x) = x P_n'(x) - P_{n-1}'(x)$ . 10

7. (a) For a Bessel function  $J_n(x)$ , prove that

$$x J_n'(x) = -n J_n(x) + x J_{n-1}'(x). \quad 10$$

(b) Prove that for a Bessel function  $J_n(x)$ ,

$$J_{-n}(x) = (-1)^n J_n(x). \quad 10$$

8. (a) Find the Laplace transform of

$$e^{-3t}(2 \cos 5t - 3 \sin 5t). \quad 10$$

(b) Find  $L^{-1} \left( \frac{4s+5}{(s-1)^2 (s+2)} \right)$ . 10

