2021

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. Answer all questions : $2 \times 10 = 20$

(a) Define partition of an interval.

(b) Define Lower Riemann Sum and Upper Riemann Sum.

(c) State Darboux Theorem.

(d) Show that $\int_0^1 \frac{dx}{\sqrt{x(1+x^2)}}$ is convergent.

(Turn over)

(e) Define Beta function.

(f) Define Gamma function

(g) State Weierstrass's M-Test for uniform convergence.

(h) Define radius of convergence.

(i) State Abel's Theorem.

(j) Define power series.

2. State and prove necessary and sufficient condition for R-Integrability. 20

3. (a) State and prove Intermediate value theorem for integral. 10

(b) If f is continuous on [a, b] then $f \in R[a, b]$: 10

4. (a) If $f \in R[a, b]$, $g \in R[a, b]$

Then :

(i) $f \pm g \in R[a, b]$

(ii) $\int_a^b (f \pm g) = \int_a^b f \pm \int_a^b g$ 10

(b) If $f \in R[a, b]$, $g \in R[a, b]$ then $fg \in R[a, b]$ 10

5. (a) Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges or diverges according as $n < 1$ or $n \geq 1$. 10

(b) Prove that $\Gamma(n + 1) = n!$ where n is a natural number. 10

6. State and prove Cauchy's criterion for uniform convergence. 20

7. (a) Prove that the sequence $\{f_n(x)\}$ where $f_n(x) = x^{n-1}(1-x)$ converges uniformly in [0, 1]. 10

(b) If $\{f_n(x)\}$ be a sequence of continuous function on an Interval [a, b] and if $\{f_n(x)\}$ converges uniformly to f(x) on [a, b] then f(x) is continuous on [a, b]. 10

8. (a) State and prove Cauchy Hadamard Theorem. 10

(b) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{\lfloor n}$. 10

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