UG — Math (C – 410)

2021

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. Answer **all** questions of the following : $2\times10=20$

(a) Define ring with unity element.

(b) Define Division Ring.

(c) Define Null Ring.

(d) If a, b, c, d are elements of a ring then evaluate $(a+b)(c+d)$

(e) Define field of quotients.

(f) Define Ring Isomorphism.

(g) Define homomorphism of rings.

(h) Define prime ideals.

(i) Define maximal ideal.

(j) State Eisenstein criterion of irreducibility.

2. (a) Prove that a ring is without zero division if and only if the Cancellation Law holds in R. 10

(b) Every field is an integral domain but converse is not true. 10

3. (a) Prove that for a non-empty subset S of a ring R to be subring of R $\Leftrightarrow$: 10

(i) $a \in s, b \in s \Rightarrow a - b \in s$

(i) $a \in s, b \in s \Rightarrow ab \in s$

(b) Prove that the characteristic of an integral domain is either 0 or a prime number. 10

4. (a) Prove that the intersection of two ideals of R is an ideal of R. 10

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(b) Prove that the ring of integers is a principal ideal ring. 10

5. Define field of quotients of an integral domain. Show that any isomorphic integral domain have isomorphic quotients fields. 20

6. (a) Let R is a ring, S is an ideal of R. Let f be any mapping from R to R/S defined by $f(a) = S + a, \forall a \in R$ then prove that f is a homomorphism of R onto R/S. 10

(b) Let f be an isomorphism of a ring R onto ring R′. If R′ is an integral domain then prove that R is also an integral domain. 10

7. (a) If R is a ring and $f, g, h \in R[x]$, then show that : 10

(i) $f(g + h) = fg + fh$

(ii) $(g + h)f = gf + hf$

(b) If D is an integral domain then prove that D[x] is also an integral domain. 10

8. (a) Show that a polynomial domain F[x] over a field F is a principal ideal ring. 10

(b) Prove that every Euclidean ring is a principal ideal ring. 10