2022

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. Answer **all** questions of the following :

$2 \times 10 = 20$

(a) Define Metric space.

(b) Define Neighbourhood.

(c) Define Accumulation point of a set in Metric space.

(d) Define Analytic function.

(e) Which one is not correct in Metric space ?

(i) $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$

(ii) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

(iii) $\overline{A} \cap \overline{B} \subset \overline{A \cap B}$

(iv) $\overline{\overline{A}} = \overline{A}$

(f) State Cantor's Intersection theorem.

(g) Define fixed point of a bilinear transformation.

(h) Prove that the function $|Z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

(i) Write Laplace differential equation.

(j) Write the sufficient condition for $w = f(z)$ to represent a conformal mapping.

2. (a) Prove that if (E, d) is a Metric space, then (E, ρ) where ρ is defined by $\rho(x, y) = \dfrac{d(x,y)}{1 + d(x,y)}$ is also a metric space. 10

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(b) Prove that in a metric space a set is open iff it is union of open spheres. 10

3. (a) Prove that in a metric space every closed sphere is a closed set. 10

(b) Let E be a complete metric space and F be a subspace of E, then F is complete iff it is closed subset of E. 10

4. (a) State and prove Banach fixed-point theorem. 10

(b) State and prove Cauchy-Schwarz inequality. 10

5. (a) Prove that : $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$. 10

(b) If Z_1 and Z_2 are two complex numbers then prove that $|Z_1 + Z_2|^2 + |Z_1 - Z_2|^2 = 2(|Z_1|^2 + |Z_2|^2)$ and deduce that

$$|\alpha + \sqrt{\alpha^2 - \beta^2}| + |\alpha - \sqrt{\alpha^2 - \beta^2}| = |\alpha + \beta| + |\alpha - \beta|.$$ 10

6. Prove that the necessary and sufficient condition that a function $f(z) = u(x, y) + iv(x, y)$ is differentiable at any point $Z = x + iy$, then $u_x = v_y$, $u_y = -v_x$. 20

7. (a) Define Cross-ratio and prove that the cross-ratio remains invariant under bilinear transformation. 10

(b) Find the bilinear transformation which maps the points $Z_1 = 1$, $Z_2 = -i$ and $Z_3 = 2$ onto the points $w_1 = 0$, $w_2 = 2$ and $w_3 = -i$ respectively. 10

8. Prove that every bilinear transformation is angle preserving. 20