2022

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any four questions in which Q. No.1 is compulsory.

1. Answer all questions of the following : $2 \times 10 = 20$

(a) Define a vector space.

(b) State the necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V(F).

(c) Define linearly dependent and linearly independent vectors in a vector space.

(d) Define dual space of a vector space.

(e) Define a basis of a finite dimensional vector space.

(f) Define orthonormal complement of a non-empty set.

(g) Define Annihilators and transpose of a linear transformation.

(h) In an inner product space, prove that $(x, \alpha y + \beta z) = \bar{\alpha}(x, y) + \bar{\beta}(x, z)$.

(i) Define adjoint of a linear operator.

(j) Define normal operators in an inner product space.

2. Show that the set $Q\sqrt{2} = \{a + b\sqrt{2} : a, b \in Q\}$ is a vector space over set of rational numbers Q, with respect to the compositions : $(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a + c) + (b + d)\sqrt{2}$ and $\alpha(a + b\sqrt{2}) = \alpha a + \alpha b\sqrt{2} : \alpha \in Q$. 20

3. If W be a subspace of a finite dimensional vector space V(F) then prove that dim.V/W = dim. V – dim.W. 20

Contd.

4. (a) Let T be an invertible linear transformation on a vectore space V(F), then prove that $T^{-1}T = I = TT^{-1}$. 10

(b) Let $T : U \rightarrow V$ be a linear transformation from vector spaces U(F) to V(F), then Ker (T) or N(T) is a linear subspace of U. 10

5. Let V be a finite dimensional vector space over the field F and let $B = \{v_1, v_2, v_3, ..., v_n\}$ be a basis for V. Then there is a uniquely dtermined basis $B^* = \{f_1, f_2, f_3, ..., f_n\}$ such that $f_1(v_j) = o, i \neq j$ and $f_1(v_j) = 1, i = j$. 20

6. (a) State and prove Cayley – Hamilton theorem. 10

(b) Let $T : R^2 \rightarrow R^2$ be the linear operator defined by $T(x, y) = (2x + 3y, 4x - 5y)$. Find the matrix representation of T relative to the basis $B = \{\beta_1, \beta_2\} = \{(1,2), (2,5)\}$. 10

7. (a) Define inner product space and orthonormal complement of a non-empty set. 10

(b) If $S^{\perp}$ denotes the orthonormal complement of a set S in an inner product space E, then $S^{\perp}$ is a subspace of E. 10

8. Describe briefly the Gram-Schmidt orthonormalization process. 20

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