UG — Math
(D – 603 A)

2022

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. Answer the following questions : $2 \times 10 = 20$

(a) If $2 + 3i$ be one of the two roots of the equation $x^2 - 4x + 13 = 0$, then the other root is :

(i) 2

(ii) $2 + 3i$

(iii) $2 - 3i$

(iv) None of these

(b) The equation $x^3 - 4x^2 - 2x + 1 = 0$ cannot have more than :

(i) 2 positive roots

(ii) 1 negative root and 1 positive root

(iii) 1 positive root

(iv) None of these

(c) Every Polynomial equation of an odd degree has :

(i) No real root

(ii) At least one real root

(iii) All real roots

(iv) All roots imaginary

(d) If α, β, γ are roots of the equation $7x^3 + 5x + 11 = 0$, then find $\Sigma\alpha\beta$ and $\Sigma\alpha$.

(e) If α, β, γ are roots of the equation $x^3 + px^2 + qx + r = 0$, then find $\Sigma\alpha^2$.

(f) Define symmetric function.

(g) Solve the equation $x^3 + x^2 + x = 0$, given that one of its roots is $\frac{1}{2}\left(-1 + \sqrt{-3}\right)$.

Contd.

(h) Define symmetric function with example.

(i) State Sturm's theorem.

(j) Write at least two properties of Sturm's function.

2. (a) One of the root of the equation $x^3 + x^2 - x + 15 = 0$ is -3, then find the other roots. 10

(b) Prove that every equation of an odd degree has at least one real root of a Sign opposite to that of the last term. 10

3. (a) Using Descarte's rule of signs, find the nature of the roots of the equation $x^4 + 15x^2 + 7x - 11 = 0$ whose roots are in A. P. 10

(b) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ are in G. P. 10

4. Calculate the values of the symmetric functions for cubic equation $x^3 + px^2 + qx + r = 0$ whose roots are α, β, γ : 20

(a) $\Sigma\alpha^2\beta^2$

(b) $\Sigma\alpha^2\beta$

(c) $\Sigma\alpha^3$

(d) $\sum \frac{\beta^2 + \gamma^2}{\beta\gamma}$

5. (a) If α, β, γ be the roots of the equation $x^3 + px + q = 0$, Prove that : 10

$$\frac{\alpha^5 + \beta^5 + \gamma^5}{5} = \frac{\alpha^3 + \beta^3 + \gamma^3}{3} \times \frac{\alpha^2 + \beta^2 + \gamma^2}{2}$$

(b) Solve the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing the 2nd term. 10

6. Find the equation whose roots are the cubes of the roots of the equation $x^3 + px^2 + qx + r = 0$, and if α, β, γ be the roots of the equation find the values of $\Sigma\alpha^3$ and $\Sigma\alpha^3\beta^3$. 20

7. (a) Solve the equation : 10

$$x^3 - 30x - 133 = 0$$

(b) Solve the equation : 10

$$x^4 - 3x^2 - 6x - 2 = 0$$

8. (a) State and prove Newton's theorem on the sum of powers of the roots of an equation. 10

(b) Use Sturm's method to show that the equation $x^4 - 12x + 7 = 0$ has a root between 2 and 3. 10

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